FUZZY GOAL PROGRAMMING APPROACH TO SOLVE NON-LINEAR BI-LEVEL PROGRAMMING PROBLEM IN STRATIFIED DOUBLE SAMPLING DESIGN IN PRESENCE OF NON-RESPONSE

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ABSTRACT
In the present paper a multivariate stratified population is considered with unknown strata weights and an optimum sampling design is proposed in the presence of non-response to estimate the unknown population means using DSS strategy. The problem turns out to be a non-linear bi-level programming problem. Then a fuzzy goal programming approach is used to solve the non-linear bi-level programming problem. The objective function of each level decision maker is non-linear in nature and there is one linear constraint with some upper and lower bounds. To demonstrate the efficiency of the proposed approach, an illustrative numerical example is provided

Key Words: Bi-level programming, Fuzzy goal programming, Multivariate stratified population, Non-response, Optimum sampling design.

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1 INTRODUCTION

In sample surveys we often experience the problem of non-response. Non-response means that the desired information is not obtained/available for all units selected in the sample for one reason or the other. For example, if the sampling unit is an individual then the selected person may not be willing to provide the required information or he may not be at home when the interviewer called. In case of non-response the sampler has an incomplete sample data that affects the quality of estimates of the unknown population parameters. Hansen and Hurwitz [25] were first who dealt with the problem of non-response in mail surveys. They selected a preliminary sample and mailed the questionnaires to all the selected units. Non-respondents are identified and a second attempt was made by interviewing a subsample of non-respondents. They constructed the estimate of the population mean by combining the data from the two attempts and derived the expression for the sampling variance of the estimate. The optimum sampling fraction among the non-respondents is also obtained. El-Badry [19] extended the Hansen and Hurwitz's technique by sending waves of questionnaire to the non-respondents units to increase the response rate.

Khare [34] investigated the problem of optimum allocation in stratified sampling in presence of non-response for fixed cost as well as for fixed precision estimate.

The problem of optimum allocation in stratified random sampling is well known in sampling literature for a univariate population. Work is done in this respect by Cochran [14] and Sukhatme et al. [48].But when more than one characteristics are under study then it is not possible to use individual optimum allocation to each strata because allocation which is optimum for one characteristic may not be optimum for the other characteristic. There should be a positive strong correlation between the characteristics under study. Thus, usually; one has to use an optimum allocation that is optimum in 'some sense' for all the characteristics. Such an allocation is known as a compromise allocation in sampling literature. Methods for solving the problem of optimum allocation in multivariate stratified sampling .Peter and Bucher[42], Geary [23], Dalenius [15], Ghosh [24], Yates [53], Aoyama[4], Folks and Antle [20], Chatterjee [10], [11], Kokan and Khan [35], Ahsan [1], [3], Ahsan and Khan [2], Bethel [5], [6], Schittkowski [45], Chromy [13], Jahan,Khan and Ahsan [28], [29], Jahan and Ahsan [29], Khan, Ahsan and Jahan [31], Khan,Khan and Ahsan [32], [33], Singh [46], Diazandom-Gracia and cortez [17], [18], Kozak [36], [37] and many others either suggested new comprmise criterian further the existing criteria under various situations.

When some auxiliary information is available, it may be used to increase the precision of the estimate. Ige and Tripathi [27], Rao [43], Tripathi Bhal [50] and some other authors discussed the use of auxiliary information in stratified sampling using double sampling technique.

The problems of optimum allocation, where the strata weights are unknown and non-response also occurs have been studied by some authors. Okafor [40], solved the above problem for stratified population in univariate case using a double sampling strategy (DDS). The same problem was also formulated by Najmussehar and Bari [39] using dynamic programming technique to obtain a solution. A comparative study has also been done by Varshney et al. [51] by developing a goal programming to solve the problem.

In this paper we consider the problem of determining a compromise allocation in multivariate stratified random sampling, when strata weights are unknown and non-response is also present, has been studied. The strata weights are estimated using double sampling. The problem of obtaining a compromise allocation has been formulated as a non-linear bi-level programming problem.

Bilevel programming problems form an important class of optimization problems involving hierarchical decision making processes where the upper level decision maker anticipates the responses from the lower level and proceeds with optimizing its own objective. Their origin traces back to the Stackelberg competition models in economics. The upper level decision maker is called the leader's problem and that the lower level is called the follower's problem. The follower executes its policies after and in view of the decisions of the upper level decision maker. Control over the decision variables is partitioned among the levels but a decision variable of one level may affect the objective function of other level.

Then Fuzzy goal programming technique is used to solve the non-linear bi-level programming problem and obtained an integer solution directly by the optimization software LINGO.

LINGO is a user's friendly package for constrained optimization developed by LINDO System Inc. A user's guide-LINGO User's Guide (2001) is also available. For more information one can visit the site http://www.lindo.com

A numerical example is also presented to illustrate the computational details.

2 Double Sampling for Stratification in Presence of Non-Response

Let a multivariate survey be designed to estimate the number of persons suffering from certain specific diseases like Diabetes, High Blood Pressure, Cataract, Glaucoma, HIV etc. in a city having a population of size N, divided into three strata according to the family income. The information is to be obtained through mailed questionnaires. Further let the actual sizes of the strata say N_1, N_2 and N_3 be not known. In mailed questionnaire surveys usually the problem of non-response is also present. Under the above circumstances the surveyor may use the technique discussed in this manuscript.

Consider a population of size N, divided in to L non-overlapping strata of sizes $N_1, N_2, \ldots, N_L$, where $\sum_{h=1}^{L} N_h = N$. If $N_1, N_2, \ldots, N_L$ are not known in advance then the strata weights $W_h = N_h/N : h = 1,2,\ldots,L$ remain unknown. In such a situation double sampling technique may be used to estimate the unknown W_h by taking a large preliminary sample of size n', treating the population as unstratified. The units $n'_h; h = 1,2,\ldots,L$ of the sample falling in each stratum are recorded. An unbiased of W_h is given by $w_h = n'_h/n'$. Subsample of sizes $n_h = v_h n'_h; h = 1,2,\ldots,L; 0 < v_h \le 1$ is then drawn out of n'_h units using srswor from each stratum for fixed v_h. The double sampling for stratification (DSS) estimator of the population mean $\overline{Y}_j$ of the j^{th} characteristic out of p characteristics measured on each selected unit is given as

$$\overline{y}_{jds} = \sum_{h=1}^{L} w_h \overline{y}_{jh} \tag{1}$$

where $\overline{y}_{jh} = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{jhi}$ is the sample mean of j^{th} characteristic, $j = 1,2,\ldots,p$, based on n_h units for stratum h and 'ds' stands for double sampling.

The sampling variance of $\overline{y}_{jds}$ is given as

$$V\left(\overline{y}_{jds}\right) = \left(\frac{1}{n'} - \frac{1}{N}\right)S_j^2 + \frac{1}{n'}\sum_{h=1}^{L} w_h\left(\frac{1}{v_h} - 1\right)S_{jh}^2 \tag{2}$$

where $S_j^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_{ji} - \overline{Y}_j)^2$ is the population of j^{th} characteristic based on N units and $S_{jh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{jhi} - \overline{Y}_{jh})^2$ is the population variance for j^{th} characteristic based on N_h units for stratum h.

The expression (1) and (2) assume total response.

In the presence of non-response, let n_{h1} units respond at the first call and n_{h2} units denote the number of non-respondents out of n_h units. Using Hansen and Hurwitz technique, a subsample of non-respondents of size $m_{h2} = k_h^* n_{h2}; 0 < k_h^* < 1$ out of n_{h2} units is drawn and interviewed with improve method. Where k_h^* is a known constant.

For j^{th} characteristic, an unbiased estimator $\bar{y}^*_{jds}$ for $\bar{Y}_j$ based on sample means from the respondents and the non-respondents group obtained in second attempt is given as

$$\bar{y}^*_{jds} = \sum_{h=1}^{L} w_h \bar{y}^*_{jh} \tag{3}$$

where, $\bar{y}^*_{jh} = \frac{n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jm_{h2}}}{n_h}$

$\bar{y}_{jh1}$ sample mean for respondents based on n_{h1} units

$\bar{y}_{jm_{h2}}$ sample mean for the non-respondents based on m_{h2} units (second attempt)

The variance of $\bar{y}^*_{jds}$ is given as

$$V(\bar{y}^*_{jds}) = V\left(\bar{y}_{jds}\right) + \frac{1}{n'}\sum_{h=1}^{L} w_{h2}\left(\frac{1-k_h^*}{k_h^* \nu_h}\right) S^2_{jh2}$$

$$= \left(\frac{1}{n'} - \frac{1}{N}\right) S_j^2 + \frac{1}{n'}\sum_{h=1}^{L} w_h \left(\frac{1}{\nu_h} - 1\right) S^2_{jh} + \frac{1}{n'}\sum_{h=1}^{L} w_{h2}\left(\frac{1-k_h^*}{k_h^* \nu_h}\right) S^2_{jh2}$$

$$= V_j; j = 1,2,\ldots,p \tag{4}$$

where $w_{h2} = n_{h2}/n_h$ is the proportion of the non-respondents and S^2_{jh2} is the population variance of j^{th} characteristic, $j = 1,2,\ldots,p$, of the non-respondents in hth stratum.

Assuming a linear cost function the total cost of the survey may be given as

$$C = c_0 n' + \sum_{h=1}^{L} c_{h1} n_h + \sum_{h=1}^{L} c_{h11} n_{h1} + \sum_{h=1}^{L} c_{h12} m_{h2} \tag{5}$$

where c_0 is per unit cost of getting information from the preliminary sample, c_{h1} is per unit cost of making the first attempt (phase-I),

$c_{h11} = \sum_{j=1}^{p} c_{jh11}$ is the per unit cost for processing the result of all the p characteristics on the n_{h1} selected units from respondents group in the h^{th} stratum at phase-I,

$c_{h12} = \sum_{j=1}^{p} c_{jh12}$ is the per unit cost for measuring and processing the results of all the p characteristics on the m_{h2} units selected from the non-respondents group in the h^{th} stratum at the second attempt (phase-II),

c_{jh11} and c_{jh12} are the per unit costs of measuring the j^{th} characteristics at phase-I and phase-II respectively.

Since n_{h1} is not known until the first attempt has been made, the quantity $w_{h1}n_h$ may be used as its estimated value. The total expected cost $\hat{C}$ of the survey is thus given as

$$\hat{C} = c_0 n' + \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11}) n_h + \sum_{h=1}^{L} c_{h12} m_{h2} \tag{6}$$

3 Formulation of the Problem

Now if we encounter a multivariate problem where it is given that a certain character must be given priority over other characters and has control over particular sample sizes then that problem can be solved using BPP. Let us consider a multivariate problem partitioned into four strata with two characters where variance of one character is given priority over other and controls the certain strata size. In this way the multivariate problem can be solved as a bi-level programming problem. Now the formulation of the problem for phase-I where the problem is to find the optimum sizes of the subsamples $n_h; h = 1,2,\ldots,L$ which may be obtained by minimizing $V_j; j = 1,2$ for the fixed cost or by minimizing the cost for fixed variance may be given as:

$$\left.\begin{array}{c} \min_{n_1,n_2} V_1 \quad where\ n_3, n_4\ solves \\ \min_{n_3,n_4} V_2 \\ s.t. \\ \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11}) n_h + \sum_{h=1}^{L} c_{h12} m_{h2} \le (\hat{C} - c_0 n') \\ 2 \le n_h \le n_h' \\ and\ \ n_h\ integers; h = 1,2,\ldots,L \end{array}\right\} \tag{7}$$

where $V_j; j = 1,2$ are as defined in (4).

Ignoring the terms independent of n_h, minimizing V_j will be equivalent to minimize

$$Z_j(n_1, n_2, \ldots, n_L) = \frac{1}{n'}\sum_{h=1}^{L}\left(\frac{w_h n_h' S^2_{jh} + w_h((1-k_h^*)/k_h^*) n_h' S^2_{jh2}}{n_h}\right)$$

$$= \sum_{h=1}^{L} \frac{a_{jh}}{n_h}$$

$$= Z_j; j = 1,2 \tag{8}$$

where $a_{jh} = \left(\frac{w_h n_h' S_{jh}^2 + w_h((1-k_h^*)/k_h^*) n_h' S_{jh2}^2}{n'}\right)$

The cost constraint may be expressed as

$$\sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h \le \hat{C}_0; where\ \hat{C}_0 = \hat{C} - c_0 n' - \sum_{h=1}^{L} c_{h12} m_{h2}$$

Problem (7) may be restated as

$$\left.\begin{array}{ll} \min_{n_1,n_2} Z_1 \quad where\ n_3, n_4\ solves & (upper\ level) \\ \min_{n_3,n_4} Z_2 & (lower\ level) \\ \quad s.t.\ \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h \le \hat{C}_0 & \\ \quad 2 \le n_h \le n_h' & \\ \quad and\ \ n_h\ integers;\ h = 1,2,\ldots,L & \end{array}\right\} \quad (9)$$

where $Z_j; j = 1,2$ are as defined in (8).

At phase-II, the problem is to work out the optimum values of m_{h2} which may be obtained by minimizing $V_j; j = 1,2$ given by (4) for given cost in (6).

Ignoring the terms independent of m_{h2} in the RHS of (4), substituting $k_h^* = m_{h2}/n_{h2}$ and $v_h = n_h/n_h'$, the problem (9) may be stated as

$$\left.\begin{array}{ll} \min_{m_{12},m_{22}} Z_1' \quad where\ m_{32}, m_{42}\ solves & (upper\ level) \\ \min_{m_{32},m_{42}} Z_2' & (lower\ level) \\ \quad s.t.\ \sum_{h=1}^{L} c_{h12} m_{h2} \le \hat{C}_0' & \\ \quad 2 \le m_{h2} \le n_{h2} & \\ \quad and\ \ m_{h2}\ integers;\ h = 1,2,\ldots,L & \end{array}\right\} \quad (10)$$

where $Z_j'; j = 1,2$ are the function of $m_{h2}; h = 1,2,\ldots,L$ given by

$$Z_j'(m_{12}, m_{22}, \ldots, m_{L2}) = \frac{1}{n'} \sum_{h=1}^{L} \frac{w_{h2} n_{h2} n_h' S_{jh2}^2}{m_{h2} n_h}$$

$$= \sum_{h=1}^{L} \frac{b_{jh}}{m_{h2}}; j = 1,2$$

$b_{jh} = \frac{w_{h2} n_{h2} n_h' S_{jh2}^2}{n' n_h}$ and $\hat{C}_0' = \hat{C} - c_0 n' - \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h$.

4 The Solution Procedure by Using Fuzzy Goal Programming

We now formulate the fuzzy programming model of NLBLPP by transforming the objective functions Z_1 and Z_2 into fuzzy goals by means of assigning an imprecise aspiration level to each of them. Let Z_1^* and Z_2^* be the optimal solutions of the objective functions of ULDM and LLDM respectively when calculated in isolation subject to the system constraints.

Then the fuzzy goals appear in the form:

$Z_1 \gtreqqless Z_1^*$ and $Z_2 \gtreqqless Z_2^*$

Using the individual best solutions, we formulate a payoff matrix as follows:

$$\begin{bmatrix} & Z_1(\bar{n}) & Z_2(\bar{n}) \\ n_h^{(1)} & Z_1(n_h^{(1)}) & Z_2(n_h^{(1)}) \\ n_h^{(2)} & Z_1(n_h^{(2)}) & Z_2(n_h^{(2)}) \end{bmatrix}, \text{where } h = 1,2,\ldots,L$$

where $n_h^{(1)}$ & $n_h^{(2)}$ are the individual optimal points of the objective functions of ULDM and LLDM.

The maximum value of each column gives the upper tolerance limit for the objective functions Z_1 and Z_2. The minimum value of each column gives lower tolerance limit for the objective functions respectively.

The objective value, which is equal to or larger than Z_1^* should be absolutely satisfactory to ULDM. Similarly, the objective values, which is equal to or larger than Z_2^* should be absolutely satisfactory to LLDM. If the individual best solutions are identical, then a satisfactory optimal solution of the system is reached. However, this situation arises rarely because the objective of ULDM and LLDM is conflicting in general.

The non-linear membership function $\mu_1(\bar{n})$ corresponding to the objective function $Z_1(\bar{n})$ of the ULDM can be formulated as:

$$\mu_1(\bar{n}) = \begin{cases} 0, & \text{if } Z_1(\bar{n}) \ge Z_1^U(\bar{n}) \\ \frac{Z_1^U(\bar{n}) - Z_1(\bar{n})}{Z_1^U(\bar{n}) - Z_1^L(\bar{n})}, & \text{if } Z_1^L(\bar{n}) \le Z_1(\bar{n}) \le Z_1^U(\bar{n}) \\ 1, & \text{if } Z_1(\bar{n}) \le Z_1^L(\bar{n}) \end{cases}$$

Here $Z_1^U(\bar{n})$ and $Z_1^L(\bar{n})$ is respectively the upper and lower tolerance limits of the fuzzy objective goal for ULDM.

Similarly, the non-linear membership function $\mu_2(\bar{n})$ corresponding to the objective function $Z_2(\bar{n})$ of the LLDM can be formulated as:

$$\mu_2(\bar{n}) = \begin{cases} 0, & \text{if } Z_2(\bar{n}) \ge Z_2^U(\bar{n}) \\ \frac{Z_2^U(\bar{n}) - Z_2(\bar{n})}{Z_2^U(\bar{n}) - Z_2^L(\bar{n})}, & \text{if } Z_2^L(\bar{n}) \le Z_2(\bar{n}) \le Z_2^U(\bar{n}) \\ 1, & \text{if } Z_2(\bar{n}) \le Z_2^L(\bar{n}) \end{cases}$$

Here $Z_2^U(\bar{n})$ and $Z_2^L(\bar{n})$ is respectively the upper and lower tolerance limits of the fuzzy objective goal for LLDM.

Now the problem of phase-I reduces to

$$\left.\begin{aligned} &\min \mu_1(\bar{n}) \\ &\min \mu_2(\bar{n}) \\ &s.t.\ \sum_{h=1}^{L}(c_{h1}+w_{h1}c_{h11})n_h \le \hat{C}_0 \\ &\qquad 2 \le n_h \le n_h' \\ &and\ \ n_h\ integers; h=1,2,\ldots,L \end{aligned}\right\} \quad (11)$$

4.1 Linearization of the non-linear membership functions by first order Taylor Series

Let $\bar{n}_h^{(1)*}$ and $\bar{n}_h^{(2)*}$ be the individual best solutions of the non-linear membership functions $\mu_1(\bar{n})$ and $\mu_2(\bar{n})$ subject to the constraints. Now, we transform the non-linear membership functions $\mu_1(\bar{n})$ and $\mu_2(\bar{n})$ into equivalent linear membership functions at individual best solution point by first order Taylor series as follows:

$$\mu_1(\bar{n}) \cong \mu_1(\bar{n}_h^{(1)*}) + (n_1 - n_1^{(1)*})\frac{\partial}{\partial n_1}\mu_1(\bar{n}_h^{(1)*}) + \cdots$$

$$+(n_L - n_L^{(1)*})\frac{\partial}{\partial n_L}\mu_1(\bar{n}_h^{(1)*}) = \xi_1(\bar{n})$$

$$\mu_2(\bar{n}) \cong \mu_2(\bar{n}_h^{(2)*}) + (n_1 - n_1^{(2)*})\frac{\partial}{\partial n_1}\mu_2(\bar{n}_h^{(2)*}) + \cdots$$

$$+(n_L - n_L^{(2)*})\frac{\partial}{\partial n_L}\mu_2(\bar{n}_h^{(2)*}) = \xi_2(\bar{n})$$

4.2 FGP model of NLBLPP

The NLBLPP represented by (11) reduces to the following problem

$$\left.\begin{aligned} &\min \xi_1(\bar{n}) \\ &\min \xi_2(\bar{n}) \\ &s.t.\ \sum_{h=1}^{L}(c_{h1}+w_{h1}c_{h11})n_h \le \hat{C}_0 \\ &\qquad 2 \le n_h \le n_h' \\ &and\ \ n_h\ integers; h=1,2,\ldots,L \end{aligned}\right\} \quad (12)$$

The maximum value of a membership function is unity (one), so for the defined membership functions in (12), the flexible membership goals having the aspiration level unity can be presented as:

$$\xi_1(\bar{n}) + \delta_1 = 1$$
$$\xi_2(\bar{n}) + \delta_2 = 1$$

Here $d_1^+ \ge 0, d_2^+ \ge 0$ are the over deviational variables.

Then our fuzzy goal programming model for phase-I is:

$$\left.\begin{aligned} &minimize\ \delta_1+\delta_2 \\ &\xi_1(\bar{n}) + \delta_1 = 1 \\ &\xi_2(\bar{n}) + \delta_2 = 1 \\ &\sum_{h=1}^{L}(c_{h1}+w_{h1}c_{h11})n_h \le \hat{C}_0 \\ &2 \le n_h \le n_h' \\ &\delta_1 \ge 0 \\ &\delta_2 \ge 0 \\ &and\ \ n_h\ integers; h=1,2,\ldots,L \end{aligned}\right\}$$

Similarly we can formulate the fuzzy goal programming model for phase-II.

5 FGP Algorithms for NLBLPP

From the discussion of the previous section, the FGP algorithm for solving NLBLPP can be outlined as given below:

Step 1: Find the individual best solution of objective function for the levels subject to the system constraints.

Step 2: Formulate the payoff matrix. Then define upper and lower tolerance limits of each objective function.

Step 3: Construct non-linear membership function $\mu_1(\bar{n})$ corresponding to objective function $Z_1(\bar{n})$ of ULDM. Similarly, construct non-linear membership function $\mu_2(\bar{n})$ corresponding to the objective function $Z_2(\bar{n})$ of LLDM.

Step 4: Find the individual best solution of the non-linear membership functions $\mu_1(\bar{n})$ & $\mu_2(\bar{n})$ subject to the system constraints.

Step 5: Transform the non-linear membership functions $\mu_1(\bar{n})$ & $\mu_2(\bar{n})$ into equivalent linear membership functions $\xi_1(\bar{n})$ & $\xi_2(\bar{n})$ respectively at the individual best solution point by first order Taylor series.

Step 6: Formulate the FGP model for NLBLPP.

Step 7: Solve the FGP model using LINGO software.

Step 8: End.

6 Numerical Illustrations

For the purpose of demonstrating the use of DSS, the following numerical data are taken from Khan et al. It illustrates the proposed technique for computing the values of overall optimum allocations and the optimum sample sizes from non-respondents at phase-II. A population of size N=3850 is divided into four strata. Two characteristics are defined on each unit of the population. It is assumed that the estimation of population means of the two characteristics is of interest.

TABLE 1
DATA FOR FOUR STRATA AND TWO CHARACTERISTICS

h	w_h	S_{1h}^2	S_{2h}^2	v_h	k_h^*	c_{h1}	c_{h11}	c_{h12}
1	0.32	4817.72	8121.15	0.4	0.5	1	2	3
2	0.21	6251.26	7613.52	0.5	0.6	1	3	4
3	0.27	3066.16	1456.40	0.6	0.7	1	4	5
4	0.20	6207.25	6977.72	0.65	0.75	1	5	6

TABLE 2
SUBDIVIDED DATA AS RESPONDENT AND NON-RESPONDENT GROUPS FOR FOUR STRATA WITH TWO CHARACTERISTICS

h	Group	S_{1h}^2	S_{2h}^2	w_{hk} k=1,2
1	Respondent	2218.74	4318.28	w_{11}=0.70
	Non-respondent	1908.37	2557.62	w_{12}=0.30
2	Respondent	4056.75	5067.26	w_{21}=0.80
	Non-respondent	3541.23	3984.85	w_{22}=0.20
3	Respondent	2785.15	957.56	w_{31}=0.75
	Non-respondent	1677.65	877.13	w_{32}=0.25
4	Respondent	5015.17	3085.78	w_{41}=0.72
	Non-respondent	2156.52	2756.62	w_{42}=0.28

Table 1 shows the available information. Each stratum is further subdivided into respondent and non-respondent groups as given in table 2. It is assumed that v_h and k_h^* are known and the preliminary sample size $n' = 1000$.

In the last column of the table 2 k=1 for respondent group and k=2 for non-respondent group. Further let the total amount available for the survey be C=3,000 units. Out of these 3,000 units 750 units are earmarked for the preliminary sample of size n', 1,900 units are earmarked for phase-I and 350 units are earmarked for phase-II.

Using estimated values of strata weights and the size of selected preliminary sample, the values of $n_h' = w_h n'; h = 1,2,\dots,L$ are obtained as

$$n_1' = 320, n_2' = 210, n_3' = 270, n_4' = 200 \text{ with } \sum_{h=1}^{4} n_h' = 1{,}000.$$

After substituting the values from tables 1 and 2, the NLBLPP (9) for the first phase becomes

$$\left.\begin{array}{l} \min Z_1 = \frac{676.53805}{n_1} + \frac{374.83501}{n_2} + \frac{272.05508}{n_3} + \frac{288.54504}{n_4} \\ \min Z_2 = \frac{1077.13728}{n_1} + \frac{447.33203}{n_2} + \frac{131.54568}{n_3} + \frac{330.56571}{n_4} \\ \text{subject to} \\ 2.4n_1 + 3.4n_2 + 4n_3 + 4.6n_4 \le 1900 \\ 2 \le n_1 \le 320 \\ 2 \le n_2 \le 210 \\ 2 \le n_3 \le 270 \\ 2 \le n_4 \le 200 \\ \text{and } n_h \text{ integers}; h = 1,2,\dots,4 \end{array}\right\}$$

$Z_1^* = 11.12985$ at $(219,139,107,103)$ and $Z_2^* = 12.12468$ at $(264,143,72,107)$

are the individual best solutions of both the levels which is provided by software LINGO.

Then the fuzzy goals appear as:

$Z_1(\bar{n}) \lessapprox 11.12985$ and $Z_2(\bar{n}) \lessapprox 12.12468$

$$\text{Payoff matrix} = \begin{bmatrix} 11.12985 & 12.57542 \\ 11.65909 & 12.12468 \end{bmatrix}$$

Here

$Z_1^U(\bar{n}) = 11.65909, Z_1^L(\bar{n}) = 11.12985$ and $Z_2^U(\bar{n}) = 12.57542, Z_2^L(\bar{n}) = 12.12468$

are the upper and lower tolerance limits.

The non-linear membership functions of ULDM and LLDM are

$$\mu_1(\bar{n}) = \frac{11.65909 - Z_1(\bar{n})}{11.65909 - 11.12985}$$

$$\mu_2(\bar{n}) = \frac{12.57542 - Z_2(\bar{n})}{12.12468 - 3.16025}$$

The membership function $\mu_1(\bar{n})$ is minimal at the point (219, 139, 107, 103) and membership function $\mu_2(\bar{n})$ is minimal at the point (264, 143, 72, 107) respectively.

Then, the non-linear membership functions are transformed into linear at the individual best solution point by first order Taylor polynomial series as follows:

$$\mu_1(\bar{n}) \cong 1 + (n_1 - 219) \times 0.0267 + (n_2 - 139) \times 0.0367 + (n_3 - 107) \times 0.0449 + (n_4 - 103) \times 0.0514 = \xi_1(\bar{n})$$

$$\mu_2(\bar{n}) \cong 1 + (n_1 - 264) \times 0.0343 + (n_2 - 143) \times 0.0485 + (n_3 - 72) \times 0.0563 + (n_4 - 107) \times 0.0641 = \xi_2(\bar{n})$$

Then, the FGP model for solving NLBLPP is formulated as follows:

$$\left.\begin{array}{c} minimize\ \delta_1+\delta_2 \\ s.t.\ 1+(n_1-219)\times 0.0267+(n_2-139)\times 0.0367+ \\ (n_3-107)\times 0.0449+(n_4-103)\times 0.0514+\delta_1=1 \\ 1+(n_1-264)\times 0.0343+(n_2-143)\times 0.0485+ \\ (n_3-72)\times 0.0563+(n_4-107)\times 0.0641+\delta_2=1 \\ 2.4n_1+3.4n_2+4n_3+4.6n_4\le 1900 \\ 2\le n_1\le 320 \\ 2\le n_2\le 210 \\ 2\le n_3\le 270 \\ 2\le n_4\le 200 \\ \delta_1\ge 0 \\ \delta_2\ge 0 \\ and\quad n_h\ integers;\ h=1,2,\dots,4 \end{array}\right\}$$

By solving the FGP model by software LINGO, we get the optimal solution as:

$n_1^*=279, n_2^*=107, n_3^*=70, n_4^*=127\ with\ Z_1^*=12.08651$ $and\ Z_2^*=12.52348.$

Similarly we formulate the FGP model for phase-II as given below:

$$\left.\begin{array}{c} minimize\ \delta_1+\delta_2 \\ s.t.\ 1+(m_{12}-29)\times 0.2682+(m_{22}-39)\times 0.3391+ \\ (m_{32}-17)\times 0.4121+(m_{42}-17)\times 0.4795+\delta_1=1 \\ 1+(m_{12}-34)\times 0.2758+(m_{22}-20)\times 0.3632+ \\ (m_{32}-12)\times 0.4561+(m_{42}-18)\times 0.5766+\delta_2=1 \\ 3m_{12}+4m_{22}+5m_{32}+6m_{42}\le 300 \\ 2\le m_{12}\le 73 \\ 2\le m_{22}\le 28 \\ 2\le m_{32}\le 23 \\ 2\le m_{42}\le 29 \\ \delta_1\ge 0 \\ \delta_2\ge 0 \\ and\quad m_{h2}\ integers;\ h=1,2,\dots,4 \end{array}\right\}$$

By solving the FGP model by software LINGO, we get the optimal solution as:

$m_{12}^*=44, m_{22}^*=12, m_{32}^*=6, m_{42}^*=23\ with\ Z_1^{'*}=10.01193$ $and\ Z_2^{'*}=8.852199.$

7 CONCLUSIONS

In this paper we have shown that how a stratified double sampling designs in presence of non-response can be formulated as non-linear bi-level programming problem. And then we solve the formulated problem using fuzzy goal programming and obtain the solution in minimum number of steps.

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